Influence of ballistics to diffusion transition in primary wave propagation on parametric antenna operation in granular media

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A model of parametric transmitting antenna in granular media is developed, which takes into account velocity dispersion, frequency-dependent absorption, and frequency-dependent scattering of acoustic waves in granular media. The latter process may induce a transition from the ballistics to the diffusion regime of pump (primary) high-frequency wave propagation with increasing frequency. The conditions under which the transition from ballistics to diffusion manifests itself in the change of the demodulated (rectified) low-frequency acoustic pulse profile are established. It is demonstrated that parametric low-frequency radiation contains information on both absorption and scattering of high-frequency acoustic waves.

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I. INTRODUCTION

Linear acoustic waves traditionally provide an important tool for the evaluation of granular media properties [1–3]. The diffusion of ultrasound and the ballistic to diffusive transition in acoustic wave transport have been observed both in glass beads immersed in liquids and in glass beads in air [3–9].

At the same time, various nonlinear acoustic phenomena are also applied for the characterization of granular media [10–15]. A parametric antenna emits low-frequency acoustic signals due to the demodulation of amplitude-modulated high-frequency waves in a nonlinear medium. In Fig. 1, regions of existence in the medium of high-frequency (HF) primary waves and low-frequency (LF) demodulated (rectified) waves are schematically represented. The region of spatial variation of HF primary waves plays the role of sources for the demodulated LF waves. These LF waves penetrate deeper into the medium than the HF components due to the attenuation decrease with diminishing frequency. As a consequence, in many experimental conditions, only the demodulated low-frequency waves can be detected.

Recently, preliminary experiments on the operation of parametric transmitting antenna in granular media have been reported [15–17]. The experimental results presented in Ref. [15] have been analyzed in Ref. [16] under the hypothesis of ballistic propagation of amplitude-modulated high-frequency pump waves. In Ref. [16], it is demonstrated that information on high-frequency sound absorption in a grainy medium can be extracted from the analysis of demodulated low-frequency pulses during the nonlinear parametric process. In contrast to Ref. [15], the estimates presented in Ref. [17] provide strong indication that waves launched in a glass bead assemblage propagate diffusively. Consequently, in relation to the experiments presented in Refs. [15,17], the following question arises: Is it possible to extract information on the regime of high-frequency pump wave propagation from the received low-frequency demodulated signals?

In the present paper, a simple theoretical model is developed, which predicts the dependence of the profiles of parametrically excited LF acoustic pulses on the regime (ballistic/diffusive) of energy transport by HF pump wave packets. The profiles (and the spectra) of the parametric antenna signals contain, in general, information on absorption, scattering, and velocity dispersion of HF pump waves in granular media. Dependence of the LF demodulated signal on the HF primary-wave transport regime has been observed in recent experiments [18,19]. Comparison of the model developed below with these experiments is currently in progress.

II. THEORY

The goal of the analysis presented here is to understand how low-frequency acoustic waves, excited in the nonlinear rectification process of amplitude-modulated high-frequency waves, depend on the regime of HF waves propagation. Is it possible to use LF parametric antenna signals to solve the inverse problem dealing with the regime evaluation of the HF wave propagation?

To get qualitative answers to these questions, the simplest analytical model for the analysis of the demodulation process is proposed below. It is believed that this model might be

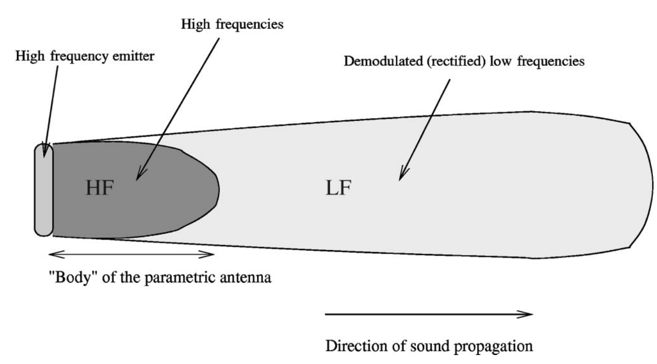

FIG. 1. Qualitative scheme of parametric antenna.

significantly improved in conjunction with experimental investigations if necessary. In other words, most of the assumptions leading to this model could be improved if needed. These simplifying assumptions are as follows.

The propagation of both HF and LF acoustic waves is treated as though it is taking place in an effective homogeneous medium. This medium is described by frequency-dependent absorption and acoustic velocity dispersion. The latter, in fact, is a manifestation of the microstructure of the granular medium, which is composed of beads having finite dimensions. Velocity dispersion increases when HF approaches the cutoff, or natural frequency [20], which is inversely proportional to the bead diameter. The fact that the beads packing is not regular (not periodic in space) is taken into account by introducing the option for the acoustic waves to be scattered without absorption. Scattering is frequency dependent. It should be clearly stated that in the following we analyze only propagative modes of the system (with frequencies below the cutoff frequency). Neither the evanescent modes nor their scattering due to irregular packing of beads is considered here.

It is assumed that scattering leads to the randomization of propagation directions of HF wave packets. Accordingly, scattering contributes to the attenuation of the initial HF acoustic wave (which propagates ballistically) and generates a scattered HF acoustic field, which, in general, may transport elastic energy quasiballistically [21], by diffusion [3–9], or it may be localized [22,23]. Here, it is assumed that scattering is accompanied by a complete loss of coherence in the initial HF acoustic waves and that the scattered HF acoustic field propagates diffusively [3–9]. The intermediate regime, where coherence is not completely lost after scattering, is not considered here [24,25]. It manifests itself by the occurrence of a signal arising after the ballistic leading front, when spatial averaging is performed on the multiple scattering sample.

We analyze here the processes initiated by launching a coherent longitudinal acoustic wave in the granular medium, which propagates mainly through the beads (solid-based mode). The air-based acoustic mode (carrying energy mainly through saturating air), which is also excited in the demodulation process [17], is neglected. For sake of simplicity, we completely ignore shear acoustic waves, which are also supported by granular media [26–28]. Shear waves may be excited by mode conversion in the scattering of compressional (longitudinal) waves. It might be expected that at sufficiently long time scales there is energy equipartition between longitudinal and shear waves in incoherent diffusive acoustic field due to multiple scattering [29,30]. This effect is currently ignored. We are also neglecting in this analysis the possible existence of microrotational (spin) waves [31,32] in granular medium. It is assumed that acoustic energy is transported by compressional waves only.

The nonlinearity of the state equation, which leads to the demodulation process, is assumed to be associated only with the nonlinearity of interbead Hertz contacts under normal loading [28,33]. Only quadratic nonlinearity is considered [16,17,34]. The hysteretic nonlinearity of sliding Hertz contacts [26,27,33] is neglected here. It is known that quadratic hysteretic nonlinearity acts as an odd-type nonlinearity and does not contribute to the demodulation of quasiharmonic acoustic waves [35].

In principle, the hysteretic nonlinearity may have a indirect influence on the demodulation process by modifying the in-depth distribution of high-frequency waves via their nonlinear absorption. However, we assume that nonlinear absorption does not provide the dominant contribution to total absorption. Consequently, in the considered here case of weakly nonlinear acoustic waves, both processes (i.e., the nonlinear absorption and the demodulation) manifest themselves at the same (second) step of successive approximations. Because of this, the cross talk between the two processes can be neglected in this approximation.

In view of the assumptions formulated above, the equation of motion for an elastic solid is written as

$$\rho_0 \frac{\partial^2 U_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j}, \tag{2.1}$$

where ρ_0 is the material density at rest, U_i are the components of the displacement vector $\vec{U}$, and σ_{ij} is the stress tensor. The description of the longitudinal waves is simply done by applying the divergence operator to Eq. (2.1),

$$\rho_0 \frac{\partial^2}{\partial t^2}(\operatorname{div}\vec{U}) = \frac{\partial^2 \sigma_{ij}}{\partial x_i \partial x_j}. \tag{2.2}$$

The elastic energy density is modeled by its expansion up to cubic terms in strain:

$$W = \rho_0 c_0^2 \left[\frac{1}{2}(\operatorname{div}\vec{U})^2 - \frac{\epsilon}{3}(\operatorname{div}\vec{U})^3 \right], \tag{2.3}$$

where c_0 represents the sound velocity and ϵ is the effective nonlinear parameter of grainy media. The stress-strain relationship corresponding to Eq. (2.3) has the form

$$\sigma_{ij} = \rho_0 c_0^2 [(\operatorname{div}\vec{U}) - \epsilon(\operatorname{div}\vec{U})^2]\delta_{ij}. \tag{2.4}$$

It should be pointed out here that in Eqs. (2.3) and (2.4), the kinematic nonlinearity is considered to be negligible in comparison with the Hertz contact nonlinearity [15,28,34]. Let the stress-strain relationship for the Hertz contact of spheres be modeled by

$$\sigma_{ij}^H = -B(-\operatorname{div}\vec{U}^H)^\kappa \delta_{ij},$$

with the constant B being proportional to the Young modulus of the beads [26–28,33,34] and where $\frac{3}{2} < \kappa < 2$ [3,9,33,36,37], both parameters depending on bead packing [36–38]. The initial stress σ_{ij}^{in} (or equivalently initial hydrostatic pressure p) is then described by

$$\sigma_{ij}^{in} = -B(-\operatorname{div}\vec{U}^{in})^\kappa \delta_{ij} = -p\,\delta_{ij}.$$

By expanding the stress-strain relationship (2.4) near the initial static strain, the super-imposed acoustic disturbance counterpart can be derived in the form

$$\sigma_{ij}=\sigma_{ij}^{H}-\sigma_{ij}^{in}\simeq\kappa B^{1/\kappa}p^{(\kappa-1)/\kappa}\Bigg[(\mathrm{div}\,\vec{U})-\frac{\kappa-1}{2}B^{1/\kappa}p^{-1/\kappa}(\mathrm{div}\,\vec{U})^{2}\Bigg],\tag{2.5}$$

where $|\mathrm{div}\,\vec{U}|\equiv|\mathrm{div}\,\vec{U}^{H}-\mathrm{div}\,\vec{U}^{in}|\ll|\mathrm{div}\,\vec{U}^{in}|$. The comparison of Eq. (2.5) with Eq. (2.4) provides the dependences of LF sound velocity and effective nonlinearity on hydrostatic pressure [15,28,34]. In particular, in the case of a perfect packing without flapping contacts [28,34,36] $\kappa=3/2$, $c_0\sim p^{1/6}$, $\epsilon\sim p^{-2/3}$. When under some circumstances, a dependence close to $c_0\sim p^{1/4}$ is experimentally observed [3], then it corresponds to $\kappa=2$ and $\epsilon\sim p^{-1/2}$. This is due to the dependence of the number of active contacts on pressure [37] or to departures at the single-contact level from the Hertzian contact law, which is in turn related to pointlike or conical asperity [37]. The nonlinear phenomena are described by successive approximations.

By substitution of the stress-strain relationship Eq. (2.4) into Eq. (2.2), the nonlinear wave equation is derived:

$$\left[\frac{\partial^2}{\partial t^2}-c_0^2\Delta+\hat{L}\right](\mathrm{div}\,\vec{U})=-c_0^2\epsilon\Delta(\mathrm{div}\,\vec{U})^2,\tag{2.6}$$

where a linear integro-differential operator $\hat{L}$ is formally added in order to account for possible velocity dispersion, absorption, and scattering of sound waves. Following a traditional approach [39], the solution of Eq. (2.6) is presented as a superposition of the primary HF acoustic field (denoted by $\vec{U}_\omega$) and the secondary LF acoustic field (denoted by $\vec{U}_\Omega$), $\vec{U}=\vec{U}_\Omega+\vec{U}_\omega$. The separation of equations for $\vec{U}_\omega$ and $\vec{U}_\Omega$ is achieved by averaging over the HF wave period, additionally neglecting higher harmonic excitation in the HF field [39],

$$\left[\frac{\partial^2}{\partial t^2}-c_0^2\Delta+\hat{L}\right](\mathrm{div}\,\vec{U}_\omega)=0,\tag{2.7}$$

$$\left[\frac{\partial^2}{\partial t^2}-c_0^2\Delta\right](\mathrm{div}\,\vec{U}_\Omega)=-c_0^2\epsilon\Delta\langle(\mathrm{div}\,\vec{U}_\omega)^2\rangle.\tag{2.8}$$

Equation (2.7) describes ballistic propagation, absorption, and scattering of HF amplitude-modulated acoustic waves launched by an emitter. Equation (2.8) describes the excitation of LF acoustic waves due to a nonlinear demodulation process. We use notation $\langle\cdots\rangle$ for averaging over the HF wave period. Note that for simplicity, dispersion, absorption, and scattering of LF waves are currently neglected. Comparison of the right-hand side (rhs) of Eq. (2.8) with the presentation of the elastic potential energy density in Eq. (2.3) demonstrates that the rhs may be approximated by $-(2\epsilon/\rho_0)\Delta\langle W_\omega\rangle$, where W_ω denotes the potential energy of the HF acoustic field. Taking into account that in the acoustic wave packets, potential energy is equal to kinetic energy (in an average over a wave period), and assuming plane one-dimensional geometry, we rewrite Eq. (2.8) as

$$\left[\frac{\partial^2}{\partial t^2}-c_0^2\frac{\partial^2}{\partial x^2}\right]U_\Omega=-\frac{\epsilon}{\rho_0}\frac{\partial}{\partial x}\langle\overline{W}_\omega\rangle,\tag{2.9}$$

where U_Ω denotes the x component of LF particle displacement and $\langle\overline{W}_\omega\rangle=2\langle W_\omega\rangle=\rho_0\langle(\partial U_\omega/\partial t)^2\rangle\equiv\rho_0\langle V_\omega^2\rangle$ denotes the total energy density of HF field (U_ω and V_ω are displacement and particle velocity in the HF acoustic field, respectively). In accordance with Eq. (2.9), the demodulation process takes place in the regions of HF energy density spatial variation.

The equation for energy transport by a plane HF acoustic wave packet, which follows from Eq. (2.7), is presented in the form

$$\frac{\partial}{\partial t}\langle W_{\omega b}\rangle+c_g(\omega)\frac{\partial}{\partial x}\langle W_{\omega b}\rangle+\frac{1}{\tau(\omega)}\langle W_{\omega b}\rangle=0,\tag{2.10}$$

where $c_g(\omega)$ denotes the group velocity of the acoustic wave at carrier frequency ω and $\tau(\omega)$ is the characteristic time of frequency-dependent attenuation, which incorporates both absorption [characterized by the time $\tau_a(\omega)$] and scattering [characterized by the time $\tau_s(\omega)$], with $\tau^{-1}(\omega)=\tau_a^{-1}(\omega)+\tau_s^{-1}(\omega)$. Note that the possible broadening of the HF wave packet, caused by the dispersion of group velocity, is currently neglected in Eq. (2.10). Index b in Eq. (2.10) indicates that this equation is valid only for ballistically propagating HF wave packets. We assume that scattered wave packets propagate diffusively and energy transport by scattered HF waves may be described by a diffusion equation

$$\frac{\partial}{\partial t}\langle W_{\omega d}\rangle=D_\omega(\omega)\frac{\partial^2}{\partial x^2}\langle W_{\omega d}\rangle-\frac{1}{\tau_a(\omega)}\langle W_{\omega d}\rangle+\frac{1}{\tau_s(\omega)}\langle W_{\omega b}\rangle,\tag{2.11}$$

where index d is used to denote acoustic energy density, which is scattered from a coherent HF acoustic field, $D_\omega(\omega)$ denotes the diffusivity of energy carrying wave packets. The diffusion coefficient is $D_\omega=c_e l^*/3$, where c_e is the energy velocity, which is defined as the ratio of energy flux to the energy density, and l^* is the transport mean free path (or the distance the waves must propagate until their direction is randomized) [4,7,40]. There are indications that in strongly scattering media, energy velocity c_e slightly differs from group velocity c_g [7]. For the purpose of current analysis, the diffusion coefficient is written in the form

$$D_\omega\equiv d\,c_g^2(\omega)\tau_s(\omega),\tag{2.12}$$

where $d\sim1$ is a nondimensional constant. Clearly Eqs. (2.10) and (2.11), which model transition from ballistics to diffusion by a single "relaxation" time [scattering time $\tau_s(\omega)$], significantly oversimplify the reality, in particular, by completely ignoring intermediate coherent effects [21,24,25]. However, it is believed that the model enables to correctly depict the basic qualitative features of the effect under consideration. The proposed model may be improved afterwards by comparison with available fine experimental data.

The boundary conditions for the energy flux at the acoustic radiator located in plane $x=0$ are

$$c_g(\omega)\langle W_{\omega b}\rangle = I_\omega f(t/\tau_m),$$

$$D_\omega \frac{\partial}{\partial x}\langle W_{\omega d}\rangle = 0, \tag{2.13}$$

where I_ω denotes the intensity of HF coherent acoustic waves launched by the radiator in a granular medium and where $f(t/\tau_m)$ describes amplitude modulation in the HF wave packet (τ_m is the characteristic modulation time). The second boundary condition for Eq. (2.11),

$$\langle W_{\omega d}(x\to\infty)\rangle \to 0, \tag{2.14}$$

reflects the decrease of acoustic energy carried by HF waves due to their absorption. The solutions of Eqs. (2.10) and (2.11) in the frequency domain, subjected to boundary conditions (2.13) and (2.14), are

$$\langle \tilde{W}_{\omega b}\rangle = \frac{I_\omega}{c_g(\omega)}\tilde{f}(\Omega)\exp\left[-\frac{1}{c_g(\omega)}\left(\frac{1}{\tau(\omega)} - i\Omega\right)\right], \tag{2.15}$$

$$\langle \tilde{W}_{\omega d}\rangle = \frac{I_\omega}{c_g(\omega)D_\omega \tau_s(\omega)}\tilde{f}(\Omega)\frac{1}{\dfrac{1}{c_g^2(\omega)}\left(\dfrac{1}{\tau(\omega)} - i\Omega\right)^2 - \dfrac{1}{D_\omega}\left(\dfrac{1}{\tau_a(\omega)} - i\Omega\right)}$$
$$\times\left\{\frac{\dfrac{1}{c_g(\omega)}\left(\dfrac{1}{\tau(\omega)} - i\Omega\right)}{\sqrt{\dfrac{1}{D_\omega}\left(\dfrac{1}{\tau_a(\omega)} - i\Omega\right)}}\exp-\left[\sqrt{\frac{1}{D_\omega}\left(\frac{1}{\tau_a(\omega)} - i\Omega\right)}x\right] - \exp\left[-\frac{1}{c_g(\omega)}\left(\frac{1}{\tau(\omega)} - i\Omega\right)x\right]\right\}, \tag{2.16}$$

where

$$\langle \tilde{W}_{\omega b,d}\rangle \equiv \int_{-\infty}^{+\infty}\langle W_{\omega b,d}\rangle e^{i\Omega t}dt,$$

$$\tilde{f}(\Omega) \equiv \int_{-\infty}^{+\infty} f(t)e^{i\Omega t}dt.$$

The total energy of the HF acoustic field, $\langle \overline{W_\omega}\rangle = \langle W_{\omega b}\rangle + \langle W_{\omega d}\rangle$, which is composed of ballistic and diffusive components, is substituted in the rhs of Eq. (2.9). The solution of Eq. (2.9), satisfying the boundary condition $U_\Omega(x=0)=0$ at the HF acoustic emitter and the condition of radiation, is then provided by

$$\tilde{U}_\Omega \equiv \tilde{U}'_\Omega e^{-i(\Omega/c_0)x} \equiv \tilde{U}_{\Omega b} + \tilde{U}_{\Omega d}$$
$$\cong \frac{\epsilon I_\omega}{\rho_0 c_0^2 c_g(\omega)}\tilde{f}(\Omega)\frac{\dfrac{1}{c_g(\omega)}\left(\dfrac{1}{\tau(\omega)} - i\Omega\right)}{\dfrac{1}{c_g^2(\omega)}\left(\dfrac{1}{\tau(\omega)} - i\Omega\right)^2 + \dfrac{\Omega^2}{c_0^2}}$$
$$\times\left\{1 + \frac{1}{\tau_s(\omega)\left(\dfrac{1}{\tau_a(\omega)} + \dfrac{D_\omega\Omega^2}{c_0^2} - i\Omega\right)}\right\}, \tag{2.17}$$

where $\tilde{U}_\Omega \equiv \int_{-\infty}^{+\infty}U_\Omega e^{i\Omega t}dt$. In Eq. (2.17), $\tilde{U}'_\Omega$ denotes the spectrum of the LF parametrically excited acoustic pulse in the coordinate system moving at LF sound velocity (co-propagating coordinate frame). The solution (2.17) is valid outside the "body" (see Fig. 1) of the parametric antenna, which [in accordance to Eq. (2.16)] is limited in space by inequalities $x \leqslant \max\{c_g(\omega)\tau(\omega), \sqrt{D_\omega \tau_a(\omega)}\}$. There are two contributions to the parametric signal [denoted in Eq. (2.17) by $\tilde{U}_{\Omega b}$ and $\tilde{U}_{\Omega d}$] appearing from the demodulation of ballistic and diffusive HF acoustic fields, respectively. It is the first term in curly brackets that corresponds to the contribution from the ballistic HF field. Consequently Eq. (2.17) can be rewritten as

$$\tilde{U}'_\Omega = \tilde{U}_{\Omega b} + \tilde{U}_{\Omega d} = \tilde{U}_{\Omega b}\left\{1 + \frac{\dfrac{1}{\tau_s(\omega)}}{\dfrac{1}{\tau_a(\omega)} + \dfrac{D_\omega\Omega^2}{c_0^2} - i\Omega}\right\}. \tag{2.18}$$

The derived solution Eq. (2.17) demonstrates that the spectrum of the demodulated signal contains information on dispersion, absorption, scattering, and diffusion of the HF acoustic field. The question arises as to how sensitive is the demodulated signal to different parameters [such as $\tau_a(\omega)$ and $\tau_s(\omega)$, for example] and how these parameters might be extracted experimentally.

III. ANALYSIS

We start the analysis of the parametric antenna signal from its part $\tilde{U}_{\omega b}$, excited via the demodulation of ballistically propagating HF waves.

A. Demodulation of ballistically propagating waves in dispersive medium

Introducing notations

$$\tau_\pm \equiv \tau\left(1 \pm \frac{c_g(\omega)}{c_0}\right),$$

$$V_0 \equiv \frac{\epsilon I_\omega}{2\rho_0 c_0^2} \tag{3.1}$$

for the characteristic times and characteristic amplitude of particle velocity in the LF acoustic wave, the spectrum of particle velocity $\tilde{V}_{\Omega b} \equiv -i\Omega \tilde{U}_{\Omega b}$ is written in the form

$$\tilde{V}_{\Omega b} = V_0 \tilde{f}(\Omega)\tau(\omega)\left[\frac{1}{\tau_-}\frac{\Omega}{\left(\Omega + i\dfrac{1}{\tau_-}\right)} + \frac{1}{\tau_+}\frac{\Omega}{\left(\Omega + i\dfrac{1}{\tau_+}\right)}\right]. \tag{3.2}$$

The "splitting" of the characteristic attenuation time $\tau(\omega)$ into two characteristic times $\tau_\pm$ [Eq. (3.1)] is caused by the presence of sound velocity dispersion in granular medium [$c_g(\omega) \neq c_0$] [40,41]. In the current paper we assume for definiteness that $\partial c_g(\omega)/\partial\omega < 0$ in the whole frequency range of interest [40,41]. It is expected that τ_+ approaches τ_- when ω tends to the cutoff frequency ω_c, and consequently, $c_g(\omega) \to 0$. In the present paper we clearly limit ourselves to the analysis of propagating HF acoustic modes ($\omega < \omega_c$), leaving the case of evanescent modes for some future research.

Assuming that, in general, inequality $1/\tau_+ \ll 1/\tau_-$ may hold, it is possible to divide the spectrum Eq. (3.2) by strong inequalities into three characteristic regions:

$$\tilde{V}_{\Omega b} \cong V_0 \tilde{f}(\Omega) \times \begin{cases} 2(-i\Omega\tau) & \text{if } \Omega \ll \dfrac{1}{\tau_+} \leqslant \dfrac{1}{\tau_-}, \\ (-i\Omega\tau) & \text{if } \dfrac{1}{\tau_+} \ll \Omega \ll \dfrac{1}{\tau_-}, \\ \dfrac{\tau}{\tau_-} & \text{if } \dfrac{1}{\tau_+} \leqslant \dfrac{1}{\tau_-} \ll \Omega. \end{cases} \tag{3.3}$$

The intermediate regime $1/\tau_+ \ll \Omega \ll 1/\tau_-$ is obviously absent when inequality $1/\tau_+ \ll 1/\tau_-$ does not hold. If we assume that $f(\theta) \equiv f(t/\tau_m)$ is a hat-type modulation function with a characteristic duration τ_m, then in the time domain the characteristic regimes in Eq. (3.2) can be described as

$$V_{\Omega b} \cong V_0 \times \begin{cases} 2\left(\dfrac{\tau}{\tau_m}\right)\dfrac{\partial f}{\partial\theta} & \text{if } \tau_m \gg \tau_+ \geqslant \tau_-, \\ \left(\dfrac{\tau}{\tau_m}\right)\dfrac{\partial f}{\partial\theta} & \text{if } \tau_+ \gg \tau_m \gg \tau_-, \\ \dfrac{\tau}{\tau_-} f & \text{if } \tau_+ \geqslant \tau_- \gg \tau_m. \end{cases} \tag{3.4}$$

In accordance with Eq. (3.4), the profile of the demodulated LF acoustic pulse essentially depends on the relation between the characteristic duration of the HF wave packet $\tau_m \sim 1/\Omega$ and τ_-. The profile $V_{\Omega b}(\theta)$ of particle velocity in the LF acoustic pulse reproduces the envelope $f(\theta)$ of the HF wave packet if $\tau_- \gg \tau_m$, and it is proportional to the derivative of the envelope $\partial f(\theta)/\partial\theta$ if $\tau_m \gg \tau_-$. Clearly the third regime in Eq. (3.4) does not exist in the absence of velocity dispersion [when $c_g(\omega) = c_0$, $\tau_- = 0$ and therefore inequality $\tau_- \gg \tau_m$ never holds], for example, in underwater parametric antenna in the absence of gas bubbles [39]. Consequently, the significant dependence of the demodulated signal profile on parameters of the granular medium predicted by Eq. (3.4) is an important manifestation of the dispersive properties of the granular medium.

The experimental observation of this effect [18,19] can be used to extract information on the parameters of the granular medium. Sufficient conditions to observe the transformation from $V_{\Omega b} \sim \partial f(\theta)/\partial\theta$ to $V_{\Omega b} \sim f(\theta)$ may be formulated as follows. Let us assume that $\tau_a \ll \tau_s$. From the analysis of the $\tilde{V}_{\Omega d}$ contribution of the diffusive acoustic field to the total signal, presented later in this paper, it follows that under the condition $\tau_a \ll \tau_s$ this contribution is completely negligible ($|\tilde{V}_{\Omega d}| \ll |\tilde{V}_{\Omega b}|$) at all frequencies. For a qualitative analysis we associate $\tau_s(\omega)$ with Rayleigh-type scattering [$\tau_s(\omega) \sim 1/\omega^4$] [3] and attribute a linear dependence on frequency to sound absorption $\tau_a(\omega) \sim 1/|\omega|$ [16,28,42,43]. Then, it is expected that inequality $\tau_a(\omega_{in}) \ll \tau_s(\omega_{in})$ holds for an initial HF acoustic wave if the initial frequency ω_{in} is sufficiently far from the cutoff frequency ω_c ($\omega_{in} \ll \omega_c$). At this frequency, dispersion is expected to be weak $c_g(\omega) \simeq c_0$ and, consequently, $\tau_-(\omega_{in}) \simeq \tau_a(\omega_{in})[1 - c_g(\omega_{in})/c_0] \ll \tau_a(\omega_{in})$. Then by choosing a sufficiently high number n of HF wave periods in the initial HF wave packet, $\tau_m^{in} \equiv n(2\pi/\omega_{in})$, it is always possible to achieve $\tau_m^{in} \gg \tau_-(\omega_{in})$. If the carrier frequency ω (keeping n fixed) is then increased, the modulation time τ_m diminishes while the characteristic time τ_- increases. In fact, if we use for an estimation, the dispersion relation in the periodic chain of spherical beads [41], then

$$c_g(\omega) = c_0\sqrt{1 - \left(\frac{\omega}{\omega_c}\right)^2}$$

and

$$\frac{\partial \tau_-}{\partial \omega} = \frac{\tau_a(\omega)}{|\omega|} \frac{1}{\sqrt{1-\left(\frac{\omega}{\omega_c}\right)^2}} \left(1-\sqrt{1-\left(\frac{\omega}{\omega_c}\right)^2}\right) > 0$$

at all frequencies. Consequently, by increasing ω, we inevitably arrive at some final frequency ω_f to the condition $\tau_m^f = n(2\pi/\omega_f) \ll \tau_-(\omega_f)$. In accordance with Eq. (3.4), the transformation of signal profile $\partial f(\theta)/\partial\theta \rightarrow f(\theta)$ should be observed around a critical frequency ω_{cr}, where $\tau_m(\omega_{cr}) \sim \tau_-(\omega_{cr}) \sim \tau_a(\omega_{cr})[1-c_g(\omega_{cr})/c_0]$ holds. Consequently the observation of this transformation provides information on the absorption and dispersion of HF acoustic waves. It should be noted that the LF profile transformation is accompanied by a significant increase in signal amplitude, which is proportional to

$$\frac{\tau(\omega_f)}{\tau_-(\omega_f)} \frac{\tau_m(\omega_{in})}{\tau(\omega_{in})} = \frac{1}{1-\frac{c_g(\omega_{in})}{c_0}} \frac{\tau_m(\omega_{in})}{\tau(\omega_{in})} \gg 1.$$

From a physical point of view, additional temporal integration of the LF signal, in the case where velocity dispersion is important, is related to the absence of synchronism between the nonlinear "source" of the demodulated signal $(\partial/\partial x)\langle W_{\omega b}\rangle$ [which propagates at velocity $c_g(\omega)$] and the excited demodulated signal $V_{\Omega b}$ itself (which propagates at velocity c_0). In fact, $\tau_-(\omega)$ provides a measure of time separation between the leading fronts of LF and HF acoustic waves, which is established before attenuation of the HF wave packet takes place [that is, at time scale of the order of attenuation time $\tau(\omega)$].

In Fig. 2, we present a transformation of the demodulated signal predicted by the solution in Eq. (3.2). The numerically obtained profiles of normalized particle velocities are plotted as a function of the reduced time $t_r = t/\tau_m$, for four values of the primary-wave frequency: $\omega = 0.1\omega_c, 0.25\omega_c, 0.4\omega_c, 0.6\omega_c$ with $\omega_c = 2\times 10^6$ rad s^{-1}. The modulation function $f(\theta)$ is a Gaussian function of duration equal to 20 periods of the HF sine wave, so that $\tau_m = 20(2\pi/\omega)$. Absorption time $\tau_a(\omega) = C/\omega$ is chosen to satisfy the initial inequality $\tau_m = 20(2\pi/\omega_{in}) \gg \tau_-(\omega_{in}) = (C/\omega_{in})(1-\sqrt{1-\omega_{in}^2/\omega_c^2})$ and the final one $\tau_m = 20(2\pi/\omega_f) \ll \tau_-(\omega_f) = (C/\omega_f)(1-\sqrt{1-\omega_f^2/\omega_c^2})$. Considering $\omega_{in} = 0.1\omega_c$ and $\omega_f = 0.6\omega_c$, it gives the value of the constant $C = 100$. For the primary-wave frequency $\omega_{in} = 0.1\omega_c$, the profile in the dashed thin line is proportional to the first derivative of the modulation function $\partial f(\theta)/\partial\theta$. When increasing the primary-wave frequency, keeping the number n of periods of the HF sine wave constant, we observe a transition from $\partial f(\theta)/\partial\theta$ to $f(\theta)$: for a frequency of the primary wave $\omega = 0.25\omega_c$, continuous thin line; and $\omega = 0.4\omega_c$, dashed thick line. When arriving around frequency $\omega = 0.6\omega_c$ (continuous thick line) the profile is proportional to $f(\theta)$. The transition from $\partial f(\theta)/\partial\theta$ to $f(\theta)$ due to the velocity dispersion is thus obtained with a less than one order of magnitude change of the primary-wave frequency.

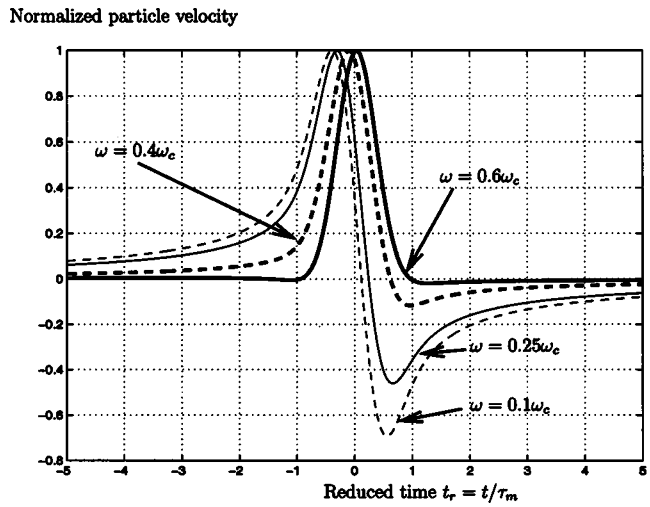

FIG. 2. Profiles of demodulated signal predicted by the solution in Eq. (3.2) for different frequencies of primary wave: $\omega = 0.1\omega_c$, dashed thin line; $\omega = 0.25\omega_c$, thin line; $\omega = 0.4\omega_c$, dashed thick line; $\omega = 0.6\omega_c$, thick line.

To conclude the analysis of the contribution to the demodulated signal from the ballistic part of the HF acoustic field, we mention that $\tilde{V}_{\Omega b}$ in Eq. (3.2) certainly contains information on the HF wave attenuation even in the frequency range $\Omega \ll 1/\tau_-$ [see Eq. (3.3)]. However, in accordance with Eq. (3.4), a transition from regime $\tau_m \gg \tau_+$ to regime $\tau_m \ll \tau_+$ is accompanied mostly by the variation of the signal amplitude without a significant envelope transformation.

B. Demodulation of diffusively propagating waves

The contribution from demodulation of the HF diffusive acoustic field $\tilde{V}_{\Omega d}$ to total signal $\tilde{V}'_\Omega = \tilde{V}_{\Omega b} + \tilde{V}_{\Omega d}$ differs from $\tilde{V}_{\Omega b}$ only by an additional spectral multiplier

$$\frac{\tau_s^{-1}(\omega)}{\left(\tau_a^{-1}(\omega) + D_\omega(\omega)\frac{\Omega^2}{c_0^2} - i\Omega\right)}$$

[see the second term in curled brackets in Eqs. (2.17) and (2.18)]. Using the notation in Eq. (2.12), we rewrite Eq. (2.18) as

$$\tilde{V}'_\Omega = \tilde{V}_{\Omega b} \times \left\{ 1 + \frac{1}{\frac{\tau_s(\omega)}{\tau_a(\omega)} + d\left(\frac{c_g(\omega)}{c_0}\right)^2 [\Omega\tau_s(\omega)]^2 - i\Omega\tau_s(\omega)} \right\}. \tag{3.5}$$

Note that in Eq. (3.5), $d \sim 1$ and $(c_g(\omega)/c_0)^2 \ll 1$. As a result it can be readily estimated that the contribution from the diffusive HF field is completely negligible [the second term in curly brackets in Eq. (3.5) is much less than 1 in its magnitude] if $\tau_a(\omega)\ll\tau_s(\omega)$ and/or $\tau_m\sim 1/\Omega\ll\tau_s(\omega)$. Both conditions have a clear physical sense: $\tilde{V}_{\Omega d}$ is negligible if the HF waves are absorbed before scattering $[\tau_a(\omega)\ll\tau_s(\omega)]$ or if scattering does not take place during the HF wave packet action on the nonlinear medium $[\tau_m\ll\tau_s(\omega)]$. In both cases, the contribution from the demodulation of ballistically propagating HF waves dominates. Thus the necessary conditions for the experimental observation of $\tilde{V}_{\Omega d}$ can be formulated as $\tau_s(\omega)\leqslant\tau_a(\omega)$ and $\tau_s(\omega)\leqslant 1/\Omega$. Assuming that generally, inequality $1/\tau_a(\omega)\ll 1/\tau_s(\omega)$ may hold [because $1/\tau_a(\omega)\sim|\omega|$, while $1/\tau_s(\omega)\sim\omega^4$], it is possible to divide by strong inequalities the spectrum Eq. (3.5) into three characteristic regions:

$$\tilde{V}'_{\Omega}=\tilde{V}_{\Omega b}\times\begin{cases}\dfrac{\tau_a(\omega)}{\tau_s(\omega)} & \text{if } \Omega\ll\dfrac{1}{\tau_a(\omega)}\ll\dfrac{1}{\tau_s(\omega)},\\[2ex] \dfrac{1}{-i\Omega\tau_s(\omega)} & \text{if } \dfrac{1}{\tau_a(\omega)}\ll\Omega\ll\dfrac{1}{\tau_s(\omega)},\\[2ex] 1 & \text{if } \dfrac{1}{\tau_a(\omega)}\ll\dfrac{1}{\tau_s(\omega)}\ll\Omega.\end{cases} \tag{3.6}$$

In the first two spectral regions in Eq. (3.6), the contribution from the diffusive HF field dominates ($|\tilde{V}_{\Omega d}|\gg|\tilde{V}_{\Omega b}|$), while in the third spectral region, the contribution from the ballistic HF field dominates ($|\tilde{V}_{\Omega d}|\ll|\tilde{V}_{\Omega b}|$). In the time domain, characteristic regimes in Eq. (3.6) can be described as

$$V'_{\Omega}=\begin{cases}\dfrac{\tau_a(\omega)}{\tau_s(\omega)}V_{\Omega b} & \text{if } \tau_m\gg\tau_a(\omega)\gg\tau_s(\omega),\\[2ex] \dfrac{\tau_m}{\tau_s(\omega)}\displaystyle\int_{-\infty}^{\theta}V_{\Omega b}(\theta')d\theta' & \text{if } \tau_a(\omega)\gg\tau_m\gg\tau_s(\omega),\\[2ex] V_{\Omega b} & \text{if } \tau_a(\omega)\gg\tau_s(\omega)\gg\tau_m.\end{cases} \tag{3.7}$$

In accordance with Eq. (3.7), there is an important difference in the profiles of the contributions to the LF signal from the ballistic and diffusive HF acoustic fields only if $\tau_a(\omega)\gg\tau_m\gg\tau_s(\omega)$. The latter inequality provides a necessary condition for the observation of the LF particle velocity profiles excited due to the demodulation of diffusively propagating HF waves and proportional to the primitive of the LF signals excited by the demodulation of ballistically propagating HF waves.

From an experimental point of view, it is important to understand (i) how the transition between the first two regimes in Eq. (3.7), where contribution from the diffusive HF field dominates, can be observed; (ii) how the transition between signals dominated by the diffusive HF field and signals dominated by the ballistic HF field [the second and the third regimes in Eq. (3.7)] can be observed. It is expected that inequality $\tau_a(\omega_{in})\gg\tau_s(\omega_{in})$ can be fulfilled at some initial frequency ω_{in} if the latter is taken sufficiently close to cutoff frequency ω_c [9] [because scattering time diminishes with frequency increase $\tau_s(\omega)\sim 1/\omega^4$ much faster than absorption time $\tau_a(\omega)\sim 1/|\omega|$]. Then under the condition $\tau_a(\omega_{in})\gg\tau_m^{in}=n_{in}(2\pi/\omega_{in})\gg\tau_s(\omega_{in})$ the profile of the demodulated signal is proportional to $\int_{-\infty}^{\theta}V_{\Omega b}(\theta')d\theta'$. A transition to profile $V_{\Omega}\sim V_{\Omega b}(\theta)$ can be achieved simply by increasing the duration τ_m of the HF wave packet by increasing n from n_i to n_f, for which inequality $\tau_m^f=n_f(2\pi/\omega_{in})\gg\tau_a(\omega_{in})$ starts to hold. Note that, because of the inequality $\tau\leqslant\min(\tau_a(\omega),\tau_s(\omega))$, in both of the considered regimes the inequality $\tau_m\gg\tau_+\geqslant\tau_-$ holds, ensuring that dispersion has no influence on the LF signal profile in this case. Consequently, in accordance with Eq. (3.7), in both regimes the contribution to V'_{Ω} from the ballistic HF waves is $V_{\Omega b}\simeq 2(\tau/\tau_m)\partial f/\partial\theta$. Thus, the transition from $\tau_a(\omega_{in})\gg\tau_m^{in}\gg\tau_s(\omega_{in})$ to $\tau_m^f\gg\tau_a(\omega_{in})\gg\tau_s(\omega_{in})$ manifests itself by the differentiation of the initial LF profile:

$$V'^{\,i}_{\Omega}\simeq V^i_{\Omega d}\simeq 2f(\theta),$$

$$V'^{\,f}_{\Omega}\simeq V^f_{\Omega d}\simeq 2\frac{\tau_a(\omega_i)}{\tau_{mf}}\frac{\partial f}{\partial\theta}. \tag{3.8}$$

The transformation of the signal profile should be observed for n_{cr}, satisfying the condition $n_{cr}(2\pi/\omega_{in})\sim\tau_a(\omega_{in})$ and is accompanied [as it follows from Eq. (3.8)] by a signal amplitude decrease, which is proportional to $\tau_m^f/\tau_a(\omega_{in})\gg 1$. The evaluation of this transformation provides information on the absorption time $\tau_a(\omega)$ of HF acoustic waves. The role of the parameter $\tau_a(\omega)/\tau_m$ in this transformation (which takes place between the regimes dominated by the demodulation of the diffusive HF acoustic field) is similar (from a physical point of view) to the role of the parameter τ_R/τ_L in the optical excitation of acoustic pulses via the generation of electron-hole plasma in semiconductors [44]. Here τ_L is the duration of the laser pulse and τ_R is the recombination time of the photogenerated electron-hole plasma.

In Fig. 3, we present the transformation of the parametric signal predicted by the solution in Eq. (3.5) for the case where the contribution from the demodulation of the scattered diffusive HF acoustic field dominates. The primary-wave frequency is fixed close to the cutoff frequency $\omega=0.9\omega_c$, with $\omega_c=2\cdot 10^6\ \mathrm{rad\,s^{-1}}$. The modulation function $f(\theta)$ is Gaussian with a characteristic duration $\tau_m=n(2\pi/\omega)$, where n is the number of the periods of HF sine wave in the signal of the primary wave. The scattering and absorption characteristic times are chosen to satisfy the inequality $\tau_a(\omega)=C_1/\omega\gg\tau_m(\omega)=n_i(2\pi/\omega)\gg\tau_s(\omega)=C_2/\omega^4$ for the frequency ω and the initial number of periods $n_i=10$. Consequently, the constants $C_1=10$ and $C_2=10^{15}\ \mathrm{rad^3\,s^{-3}}$ are fixed by the initial inequality. When increasing the number n of periods of the HF wave, the duration τ_m increases and finally becomes greater than the absorption time τ_a. This leads to the final inequality $\tau_m(\omega)\gg\tau_a(\omega)\gg\tau_s(\omega)$, ensuring for the demodulated profile, the transition from $f(\theta)$ to $\partial f(\theta)/\partial\theta$. In Fig. 3, demodulated profiles are, respectively, plotted for $n=10$, thick line; $n=80$ dashed thick line; $n=200$, thin line; and for $n=600$;

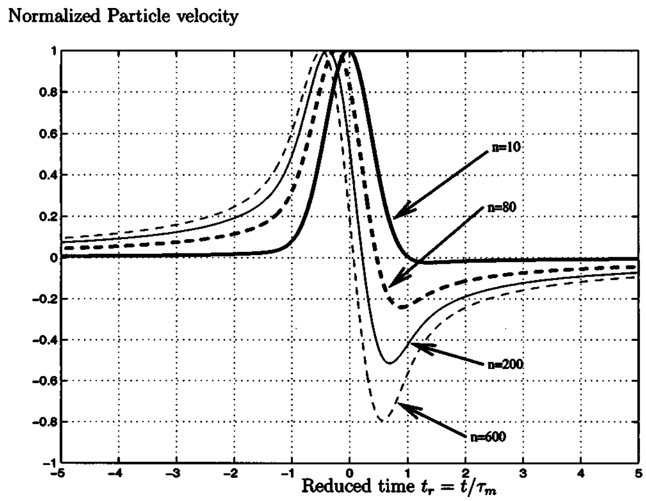

FIG. 3. Transformation of the demodulated signal predicted by the solution in Eq. (3.5) for the case where the contribution to the rectified LF signal from the demodulation of the scattered diffusive HF acoustic field dominates. The profiles are plotted as a function of the reduced time $t_r = t/\tau_m$ for four different values of HF sine wave periods in the emitted signal: $n=10$, thick line; $n=80$, dashed thick line; $n=200$, thin line; $n=600$, dashed thin line.

dashed thin line. For $n=10$, the signal is proportional to $f(\theta)$, while it is close to $\partial f(\theta)/\partial\theta$ for $n=600$.

C. Manifestation of ballistics to diffusion transition in the demodulated signal

In order to observe the transition from the regime dominated by the diffusive transport of energy by HF acoustic waves to the regime dominated by its ballistic transport, it is again possible to start from the initial condition $\tau_a(\omega_{in}) \gg \tau_m^{in} = n_{in}(2\pi/\omega_{in}) \gg \tau_s(\omega_{in})$. But this time it is useful to decrease the carrier wave frequency ω without varying the number n of HF wave periods in the wave packet ($n = n_{in} = \text{const}$). Then $\tau_a(\omega)$ and $\tau_m(\omega)$ will increase proportionally to $1/\omega$, while $\tau_s(\omega)$ will increase much faster ($\sim 1/\omega^4$), inevitably leading to the regime $\tau_a(\omega_f), \tau_s(\omega_f) \gg \tau_m^f = n_{in}(2\pi/\omega_f)$ at some final frequency $\omega = \omega_f$. In accordance with Eq. (3.7), this should cause the transformation of the LF acoustic pulse profile from $V'_\Omega = \int_{-\infty}^{\theta} V_{\Omega b}(\theta')\,d\theta'$ to $V'_\Omega \sim V_{\Omega b}(\theta)$. We recall that under the condition $\tau_a(\omega_{in}) \gg \tau_m^{in} = n_{in} \gg \tau_s(\omega_{in})$ the inequality $\tau_m^{in} \gg \tau_+(\omega_{in}) \gg \tau_-(\omega_{in})$ holds and the initial LF profile is described by the first expression in Eq. (3.8). When carrier frequency is diminished, it is expected that for the final frequency ω_f velocity dispersion is so weak that it is the inequality $\tau_+(\omega_f) \gg \tau_m^f \gg \tau_-(\omega_f)$, rather than $\tau_+(\omega_f) \gg \tau_-(\omega_f) \gg \tau_m^f$, that is realized. Consequently, in accordance with Eqs. (3.4) and (3.7) the final LF profile is described by

$$V'^f_\Omega \simeq V^f_{\Omega b} \simeq \frac{\tau(\omega_f)}{\tau_m^f} \frac{\partial f}{\partial\theta}. \tag{3.9}$$

From Eqs. (3.8) and (3.9) it follows that the transition from the regime dominated by the HF wave diffusion to the regime dominated by their ballistic propagation takes place at ω_{cr}, which can be estimated from $\tau_m^{cr} \equiv n_{in}(2\pi/\omega_{cr}) \sim \tau_s(\omega_{cr})$. This transition manifests itself in the differentiation of the parametric signal and an increase in its amplitude proportionally to $(\tau(\omega_f)/\tau_m^f) \gg 1$. The observation of this transition provides information on the characteristic time $\tau_s(\omega)$ of HF wave scattering.

In Fig. 4, we present the transformation of the demodulated signal due to the transition from the regime dominated by the HF wave diffusion to the regime dominated by their ballistic propagation. The initial condition $\tau_a(\omega_{in}) = C_1/|\omega_{in}| \gg \tau_m^{in} = n(2\pi/\omega_{in}) \gg \tau_s(\omega_{in}) = C_2/\omega_{in}^4$ and the initial frequency of the carrier wave $\omega_{in} = 0.95\omega_c$, which should be close to the cutoff frequency $\omega_c = 2\times10^6\ \text{rad s}^{-1}$, are used to determine constants $C_1 = 100$ and $C_2 = 3\times10^{18}\text{rad}^3\ \text{s}^{-3}$. Then the primary-wave frequency is decreased from $\omega_{in} = 0.95\omega_c$ to $\omega_f = 0.2\omega_c$ keeping n constant. This leads to the final condition $\tau_a(\omega_f), \tau_s(\omega_f) \gg \tau_m^f = n_{in}(2\pi/\omega_f)$. Transition from the profile $f(\theta)$ to the profile $\partial f(\theta)/\partial\theta$ is observed starting from the primary-wave frequency $\omega = 0.95\omega_c$ (thick line) and finishing with $\omega = 0.2\omega_c$ (dashed thin line), with $\omega = 0.45\omega_c$ (dashed thick line) and $\omega = 0.3\omega_c$ (thin line). Results are plotted as a function of the reduced time $t_r = t/\tau_m$, and particle velocities are normalized.

It should be mentioned, however, that the situation where in the final state $\tau_+(\omega_f) \gg \tau_-(\omega_f) \gg \tau_m^f$, in principle, can as well be achieved. The differentiation of the LF wave profile in the transition from diffusion to ballistics [predicted in Eq. (3.7)] may then be masked by the integration of the LF wave profile due to dispersion effects [predicted by Eq. (3.4)]. Obviously, it is much more difficult to extract information on the relevant parameters of the considered system when several different physical effects simultaneously influence the performance of parametric antenna. For the analysis of these situations, more precise models of the frequency dependence of sound absorption, scattering, and velocity dispersion (than those adopted in the present paper) are highly desirable.

IV. DISCUSSION

In this discussion we would like to mention that some of the predicted effects of a profile differentiation or integration, when the characteristics of pump wave (ω, τ_m, or both) are modified, may be masked by diffraction effects if the exact three-dimensional geometry of real experiments is taken into account. In fact it is well known [39,44] that diffraction may lead to additional differentiation of the pulsed signal, when its characteristic frequency diminishes (when its characteristic duration increases). To avoid cross talk between the effects discussed in the current paper and diffraction, it might be useful to detect parametric signals in their far field. In this case, diffraction can be precisely taken into account by a simple multiplication of the spectrum in Eq. (2.17) by $(-i\Omega)$ or equivalently by the differentiation of the profiles described by Eqs. (3.4) and (3.7).

Variations in the values of constants C, C_1, and C_2 of the characteristic times τ, τ_a, and τ_s from one medium to an-

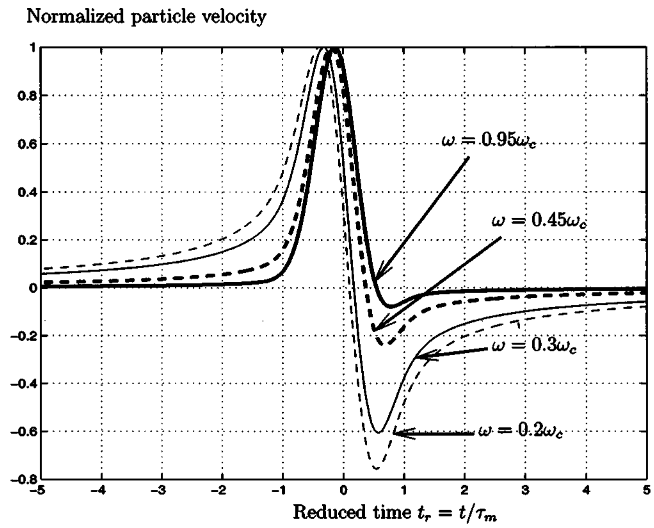

FIG. 4. Transformation of the demodulated signal predicted by Eq. (3.7) due to the transition from the regime dominated by HF wave diffusion to the regime dominated by their ballistic propagation. Profiles of demodulated signals are plotted as a function of the reduced time $t_r = t/\tau_m$ and are normalized in amplitude for primary-wave frequencies $\omega = 0.95\omega_c$, thick line; $\omega = 0.45\omega_c$, dashed thick line; $\omega = 0.3\omega_c$, thin line; $\omega = 0.2\omega_c$, dashed thin line.

other may be linked to the surface smoothness of the beads, the level of disorder in the granular assemblage, the bead material itself, the static stress supported by the column, etc. Any changes in the medium that play a role in the scattering or in the absorption of an acoustic wave of frequency ω is incorporated with the help of these constants.

This study has been devoted to the analysis of three kinds of transitions in the demodulated signal. However, it is possible to extend as well the general results to other type of transitions.

V. CONCLUSIONS

A model of parametric transmitting antenna in granular media is developed, which takes into account velocity dispersion, frequency-dependent absorption, and frequency-dependent scattering of acoustic waves in granular media. Operation of this parametric antenna is analyzed in detail in the case of the velocity difference between the HF pump waves and the demodulated LF wave, in the case of diffusion of attenuated HF primary waves, and in the case of transition from ballistics to diffusion in primary HF wave transport.

Transition in the demodulated LF profile, due to the effect of sound velocity dispersion, is observed when asynchronism between nonlinear sources created by HF waves and the LF demodulated wave has time to accumulate before the attenuation of the HF waves is taking place. The phenomenon of velocity dispersion manifests itself by the integration of the demodulated LF profile. This transition provides information on the attenuation of the ballistic component of the primary HF wave, and the velocity difference between primary HF waves and demodulated LF waves.

Differentiation of the LF profile can be obtained in the case of diffusion of the HF primary wave when increasing the duration of the initially emitted wave packet from a sufficiently low value. This transition provides information on the characteristic absorption time τ_a of primary HF wave.

The conditions on the characteristic times of scattering, absorption, and HF wave modulation are established, under which the transition from diffusion to ballistics (or vice versa) in primary HF wave propagation manifests itself in the transformation of the LF demodulated (rectified) acoustic signal. The transition from diffusion to ballistics (ballistics to diffusion) is accompanied by differentiation (integration) of the demodulated LF profile and it provides information on the characteristic scattering time τ_s of the primary HF wave.

Information on HF wave propagation is thus contained in the parametrically excited signal. However, before solving the inverse problem, it would be desirable to improve our model by taking into account precise experimental data related to linear properties of acoustic wave propagation in three-dimensional granular media (dispersion, absorption).

In order to verify theoretical predictions, we have performed preliminary experiments on parametric antenna in unconsolidated granular media [18,19]. We have observed a qualitative agreement with the model developed above, concerning predicted transitions in the demodulated LF profile. The quantitative comparison of experiments and theory is currently in progress.

The theory developed here indicates that the registration of parametrically excited low-frequency acoustic waves might be a way to get information on the propagation of HF acoustic waves, which are rapidly attenuated by absorption or scattering (and because of this it is difficult to detect them directly). The proposed method can be adapted to any medium characterized by nonsingular macroscopic nonlinearity, scattering, absorption, and dispersion. Consequently, it might find applications in the nondestructive evaluation of damaged materials.

ACKNOWLEDGMENT

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